# A PLANIMETER METHOD FOR THE DETERMINATION OF THE PERCENTAGE COMPOSITIONS OF ROCKS

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The great objection to the Rosiwal method for determining the composition of rocks lies in the nerve-racking necessity of continually reading the stage-micrometer, and the length of time required for the process. In a recent paper in the *Journal of Geology*, Shand[1] described a mechanical stage by the use of which very few readings are necessary. An instrument of this kind undoubtedly would greatly reduce the labor of computing the composition of a rock and tend to make the method more popular. Unfortunately few laboratories are likely to have such a stage; but with the equipment on hand in most institutions, measurements more rapid and possibly more accurate may be made.

Many years ago Delesse[2] determined the compositions of rocks by tracing, on oiled paper, the outline of each component as shown in polished slabs. After coloring his drawing he pasted it on tin foil, cut apart and sorted the different components, soaked off the paper, and weighed the tin foil. Later, Sollas[3] applied the method to thin sections by using a camera lucida, and Joly[4] used photomicrographs. Since Rosiwal[5] introduced his method of linear measurements in 1898, it has been the system commonly used.

[1] S. J. Shand, "A Recording Micrometer for Geometrical Rock Analysis," *Jour. Geol.*, XXIV (1916), 394–401.

[2] A. Delesse, "Procédé méchanique pour déterminer la composition des roches," *Comptes Rendus*, XXV (1847), 544–45; see also article, "Procédé méchanique pour déterminer la composition des roches," *Ann. d. Mines*, XIII (1848), 379–88.

[3] W. J. Sollas, "Contributions to a Knowledge of the Granites of Leinster," *Trans. Roy. Irish Acad., Dublin*, XXIX (1887–92), 471–73.

[4] J. Joly, "The Petrological Examination of Paving-Sets," *Proc. Roy. Dublin Soc.*, X (1903–5), 62–92.

[5] August Rosiwal, "Über geometrische Gesteinsanalysen," *Verh. d. k. k. geol. Reichsanst.*, Wein, 1898, p. 143.

The method described below is based on surface measurements. As will be shown, these are proportional to volumes in any uniform, non-banded rock, irrespective of the shape of the individual components.

Place a camera lucida over the eyepiece of the microscope and block up a drawing-board at the side so that the field of view is not too large, say four or five inches in diameter. Tilt the microscope or incline the plane of the paper until the drawing is not distorted. This may be determined by tracing the projection of a circle or square upon the paper and measuring it in several directions. If no object-glass with circle or square is at hand, place an ordinary object-scale successively vertically at the right and left of the field of view, and horizontally at the top and bottom, and lay off equal distances in the projection. If equal divisions on the scale are equal everywhere in the drawing, the plane of projection is at the proper angle or the microscope is properly tilted.

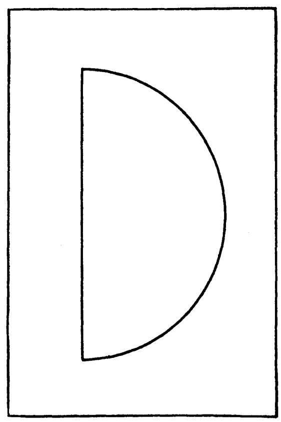

FIG. 1

Place one edge of a thin straight-edge of wood, celluloid, or cardboard on the drawing-board along the projection of the vertical cross-hair as seen through the camera lucida, and fasten it with thumb tacks. This is to serve as a guide for the stylus of a planimeter, as described below. Instead of a straight-edge, the writer uses a piece of celluloid about 1 mm. thick, in which is cut a semicircle (Fig. 1) exactly the size of the field of view ($4\frac{1}{2}$ inches in his microscope with a 31-mm. objective).

Select the objective most suitable for the particular rock section in hand; the lower the power, the better. Place a typical area in the field of view, and make on the cover-glass, on the line of the vertical cross-hair, a mark in red ink to indicate the starting-point. Place a planimeter on the drawing-board in such a position that the base will not interfere with the projection of the section as seen through the camera lucida.

Place the stylus of the planimeter at the top of the field of view and on the vertical cross-hair (Fig. 3, *A*). The writer replaces the ordinary stylus by a heavy wire bent as shown in Fig. 2, so that the end of the planimeter bar will not interfere with a view of the end of the needle point. Adjust the length of the planimeter bar so that when the stylus is moved around the semicircle (Fig. 1), that is, half the field of view, it will register exactly unity (10 or 100). This adjustment, of course, is not necessary, but by making it all readings are in percentage values without conversion; otherwise it is necessary to reduce the sum of the readings to 100.

Fig. 2

Record the reading of the planimeter or set it to zero by rotating the wheel. If the bar has not been set to read 100 for the field of view, move the stylus around the periphery of the field in the proper direction, right or left, to record additions, and read again. The difference will give the area of a single field. Again set the planimeter at zero, and, beginning at *A*, move the stylus along the straight-edge to the periphery (Fig. 3, *B*) of the first grain of the particular mineral chosen for the first determination, say biotite in granite. Trace the outline of this grain and *remember always to move the stylus in the same direction of rotation.* It will be necessary in the first readings to lift the stylus point on top of the straight-edge to record the left-hand portion of the mineral. Having returned to the point *B*, without reading the planimeter move along the straight-edge to *C* on the periphery of the next grain of biotite *lying on the line of the vertical cross-hair,* and move around it in the same way and in the same direction back to the point *C*. If the mineral of the kind being measured lies in the center of the circle (Fig. 3, *D*), it is always to be measured in

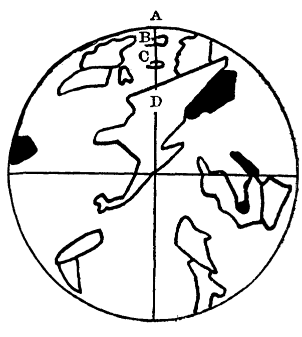


Fig. 3

the first series of readings for that kind of mineral so that it may not be forgotten.

Rotate the stage of the microscope to the left until the left margin of another grain of biotite is *just tangent* to the cross-hair. Though the point $D$ at which the stylus rested on the outline of the last mineral measured is moved to $D'$ (Fig. 4) the stylus in the projection, of course, still rests at $D$. From this position move upward or downward to the periphery $E$ of the new biotite grain tangent to the cross-hair and trace its outline in the same direction as before. Rotate the stage until another grain is tangent to the cross-hair, trace its outline, and so continue until all of the grains of biotite in the section have been measured, and the red spot again lies on the vertical cross-hair. The object of rotating the stage of the microscope only so far as the point of tangency of each new grain is to avoid confusion and the repetition or omission of some fragment. Movements up and down along the straight-edge, being plus and minus, are not recorded when the stylus again rests at $A$. When all the grains of this species of mineral have been measured and the *stylus has been returned to the point "A,"* record the planimeter reading and subtract the preceding reading from it. This gives the total area of the first mineral measured. If there are any inclusions in a mineral they may be deducted by moving them to the point of tangency and rotating the stylus around them in reverse direction.

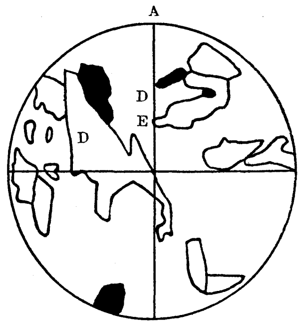


FIG. 4

Begin with the red mark on the slide again at the top of the vertical cross-hair and measure a second variety of mineral in the same manner as before, and so on until all the minerals are determined. It is usually well to measure the more pronounced or least abundant minerals first, leaving for the last the most abundant, least pronounced, or groundmass mineral. A slight error in the other readings will change the true percentage less

by leaving the most abundant mineral to the last and recording it as the difference. As a check, the last mineral should be measured and the sum of all the readings should be the same as that of the field of view.

In tracing the peripheries of the mineral grains, when they are very irregular, it is not necessary to follow their outlines absolutely; the little variations from one side to the other compensate. Furthermore, hackly ends may be averaged, long needles traced to half their length but twice their width, and so on, the results being usually more accurate than if the actual outlines are followed. Where there are a number of very small grains, as of magnetite, they may be estimated and a circle described by the stylus approximately equal to their sum. A little experience will show what may be done in the way of short cuts.

The number of fields of view to be measured in a rock slide depends upon its uniformity of texture, the mineral distribution, and the size of grain. With small grains and uniform distribution of minerals, a single field may suffice. With coarser grain or unequal distribution, it may be necessary to take as many fields as will cover the entire thin section, or it may be advisable to use several sections. If the mineral grains do not fill the field of view, or if there are pore spaces, it is only necessary to deduct the readings of these from the reading of the circle of the entire field of view and reduce the sum to 100.

The measurement of the dark minerals is easy, but the separation of the colorless minerals may give trouble. Before running the outline of one of these, it is usually well to swing off the camera lucida, insert the analyzer, and rotate the stage to various positions until the boundaries are clearly in mind; then, after replacing the camera lucida, place the edge of the mineral tangent to the vertical cross-hair as before and, with the nicols still crossed, trace the outline. The writer uses a Leitz microscope with simultaneously rotating nicols which makes it unnecessary to disturb the position of the stage; the nicols only are rotated until the mineral appears in its most pronounced color.

With crossed nicols, or when separating minerals of different refractive indices by partially closing the lower diaphragm, it

may become necessary to cut off some of the illumination from the paper. This, of course, is done by inserting the blue glass screens provided with the camera lucida. In other cases, when the light from the stage is too bright, the blue glass may be swung to that side, or, more simply, a blue glass may be placed over and parallel to the thin section and below the objective, or some of the light may be cut off by holding a finger before the mirror, by tilting it, or by partially closing the lower diaphragm. In some cases one method is best; in others, another. Sometimes the drawing or the mineral is made more distinct by simply holding the eye at a greater distance from the camera lucida.

### THE BUTTE QUARTZ MONZONITE MEASURED BY PLANIMETER

The writer uses volume percentages for his determinations instead of weight percentages. For example, a rock is half-light and half-dark to the eye when the volumes are 50–50 and not when their weight percentages are 50–50. Thus a rock half plagioclase and half magnetite by volume would have the proportions 34.4 to 65.6 by weight. In the following table, therefore,

| | Rosiwal Determinations by Four Different Students on the Same Slide Volume Percentages | | | | Average of Preceding | Average of Following | Planimeter Determinations on Four Different Areas of Same Slide as First* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | First Area | | Second Area | | Third Area | | Fourth Area | |
| Quartz | 14.80 | 17.24 | 18.07 | 17.24 | 16.84 | 18.4 | 22.9 | 22.0 | 17.5 | 18.0 | 18.1 | 17.9 | 15.1 | 14.8 |
| Orthoclase | 34.52 | 26.01 | 27.80 | 22.54 | 27.72 | 28.5 | 20.3 | 19.8 | 30.2 | 31.0 | 31.5 | 31.8 | 31.6 | 32.1 |
| Plagioclase | 34.52 | 45.50 | 38.67 | 42.46 | 40.29 | 39.9 | 46.8 | 47.2 | 36.6 | 36.0 | 34.1 | 33.9 | 42.7 | 41.8 |
| Biotite | 7.97 | 5.30 | 9.67 | 9.84 | 8.19 | 7.4 | 4.5 | 5.4 | 8.3 | 7.8 | 9.2 | 8.9 | 7.7 | 7.9 |
| Pyribole | 7.67 | 4.48 | 4.61 | 7.76 | 6.13 | 5.2 | 5.0 | 5.2 | 5.9 | 6.0 | 6.9 | 7.0 | 2.6 | 3.0 |
| Magnetite | .44 | .48 | .55 | .16 | .41 | .4 | .1 | .1 | 1.2 | 1.0 | .... | .... | .3 | .4 |
| Pyrite, etc. | .07 | .95 | .58 | ..... | .40 | .2 | .4 | .3 | .3 | .2 | .2 | .5 | .... | ..... |
| | 99.99 | 99.96 | 99.95 | 100.00 | 99.98 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

*The volume percentages by the planimeter method here given are not to be compared with the weight percentages in the paper by Johannsen and Stephenson in the preceding number of this *Journal*, since they represent a single and not typical slide; namely, the one shown in the first column of Table III in that article.

volume percentages are used. The first four columns give the Rosiwal readings made by four different students on the same slide of Butte quartz monzonite; the fifth column is their average. The sixth column is the average of eight planimeter readings on four fields of the same slide. The similarity of each pair of readings is to be noted. The differences shown by the four fields are

not due to errors of reading but to actual differences in different portions of the slide, the rock being coarse enough to show such variations. It is true that the planimeter readings were all made by one person and not by different individuals, which may account somewhat for their similarity, though clearly showing that the error cannot be great. The averages obtained by the two methods are very similar in spite of the fact that some of the students' readings are clearly in error.[1]

The length of time required for a planimeter reading of a single field of a rock of the character of the one here used is about ten minutes or less. Using four fields, a complete determination can be made with ease in three-quarters of an hour, and without any great mental strain during the process.

RELATIONSHIP BETWEEN AREAS AND VOLUMES

In the discussion of the relationship between surface and volume measurements, the mistake has commonly been made of considering this as a comparison between $d^2$ and $d^3$ to $D^2$ and $D^3$, where $d$ is the length of a side of a square representing the sum of all grains of the same mineral appearing in a section, and $D$ the side of a square representing the entire surface. As a matter of fact, in a rock with uniformly distributed particles, this is not the relationship at all, for if the mineral grains under consideration were floated to one side of the cube they would make a thin tablet, one dimension ($d_1$) depending upon the abundance of the mineral, each of the other two equal to the side of the large cube (Fig. 5, $D$). In the Rosiwal method this is the way in which the minerals are to be considered as placed, and each Rosiwal reading from west to east includes the proper proportion of the mineral. The surface value is $d_1D$, not $d_1^2$, and the volume $d_1D^2$, not $d_1^3$. If one measures areas, the mineral may be considered floated to

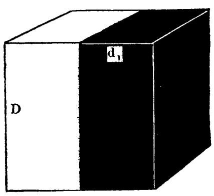

FIG. 5

[1] In explanation it must be said that the determinations here given were the students' first efforts with the Rosiwal method. The planimeter determinations are by the writer.

a vertical edge (Fig. 6) forming a vertical prism whose dimensions are $d$, $d$, and $D$; the value of $d$, of course, differing from the $d_1$ value but such that $Dd_1 = d^2$. The surface and volume values in each case, naturally, are the same. This may easily be seen by considering a solid built up of black and white cubes.

### I. CASE OF CUBICAL GRAINS, UNIFORMLY DISTRIBUTED

*a*) Assume a cube, 10 inches to a side, built up of small black and white inch cubes, uniformly distributed. If the black–white ratio is 50–50, every other block must be black. Moved to one side so that all the black cubes are together (Fig. 5), the number at the surface will be 50, distributed in a rectangle whose sides are $d_1$ and $D$ ($5 \times 10$). Moved to one edge the 50 square inches of surface would appear as a square, seven cubes on a side and one left over; in other words, $d = 7.07+$. The total number of black cubes is 500; therefore the volume is not represented by $d^3$ ($=353.5+$) but by $d^2D = 50 \times 10 = 500$. The relationship, therefore, between the black minerals appearing at the surface to the entire surface is as $\frac{d^2}{D^2}$, while the volume relation of black to white is $\frac{Dd^2}{D^3}$ and not $\frac{d^3}{D^3}$. But $\frac{d^2}{D^2}$ and $\frac{Dd^2}{D^3}$ are equal; therefore the ratios of surface and volume measurements are equal.

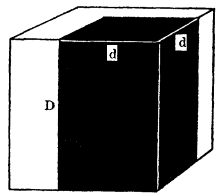


FIG. 6

*b*) If the ratio of black to white is very small, such as 1 to 10,000, it will not do to assume that the black mineral occurs as a single cube, whose volume is $\frac{1}{10}$-cubic inch, in a 10-inch cube of white, for that would vitiate the original assumption of uniform distribution of constituents. If we say that the ratio of surface to solid is as $d^2$ to $d^3$ it means that our mineral all lies in the top layer and is not uniformly distributed. It is comparable to saying that a topaz granite in which an accidental crystal of topaz occupies half the slide is half-topaz, while as a matter of fact there might be but a single crystal to a cubic foot. Consider, therefore, that the small amount of black mineral is uniformly distributed in

cubes measuring $\frac{1}{10}$ inch on a side ($d = \frac{1}{10}$ inch). If now all these small cubes be gathered along one of the vertical edges of the large cube, the portion of the black mineral of the entire block which will appear at the surface (Fig. 6) will be $d^2 = (\frac{1}{10})^2$, and the black–white surface ratio will be $\frac{d^2}{D^2} = \frac{(\frac{1}{10})^2}{10^2} = \frac{1}{10000}$. That is, the black mineral forms .01 per cent of the surface. The volume ratio of black to white is $\frac{Dd^2}{D^3} = 10\frac{(\frac{1}{10})^2}{10^2} = \frac{1}{10000}$. The volume ratio, therefore, is the same as the surface ratio.

### II. CASE OF TABULAR PLATES, UNIFORMLY DISTRIBUTED

Let $a$, $b$, and $c$ represent the ratios of length, breadth, and thickness of tabular dark constituents. If they are uniformly distributed the same number of flakes will be contained in each layer below the upper surface; consequently no matter what the thickness, $Dab$ represents the volume of the black blocks, while $D^3$ represents the large block. The ratio of surface measurements to the whole is $\frac{ab}{D^2}$, and of volumes $\frac{Dab}{D^3} = \frac{ab}{D^2}$; consequently the two are equal.

With thinner flakes there would be more layers in the large cube, and with thicker, less. But no matter what the ratio of $a$ to $b$ to $c$, the surface percentage is the same as the volume percentage, *provided the grains are equally distributed.* Of course a laminated rock, with layer of different minerals, might be so cut that one slide would contain no dark minerals and one no light. A cross-section of such a slide, however, would give correct results, for the distribution at right angles to the slide is uniform.

### III. CASE OF CLOSELY PACKED CYLINDRICAL RODS

This case is approximately the same as the preceding, for in one direction in the slide the mineral is continuous while in the other there is interstitial material. Sections should, therefore, be taken at right angles to the elongation. The surface ratio is $\frac{\pi r^2}{D^2}$, and the volume $\frac{D\pi r^2}{D^3}$. As before, the two are the same.

### IV. CASE OF SPHERES CLOSELY PACKED AND SQUEEZED TOGETHER

When spheres are closely packed and are then squeezed together they tend to assume the form of dodecahedrons. A cross-section through such a mass will give a series of triangles. If $b$ is the base of such a triangle and $h$ its height, its area is $\frac{bh}{2}$. $Nbh$ will be the area of the sum of such triangles and the ratio to the whole, $\frac{Nbh}{D^2}$. Since every parallel section will give an equal distribution of light and dark, the volume of dark mineral will be $DNbh$ and its ratio to the whole $\frac{DNbh}{D^3}$, which is the same percentage ratio as in the case of the surface measurement.

Thus, no matter what the shape of the individual mineral grains, if they are uniformly distributed the surface ratios are exactly the same as are the volume ratios; consequently surface measurements may be used to represent volume measurements in the determination of the composition of a rock.